

Effect of Astigmatism on the Tight Focusing of Azimuthally Polarized Lorentz-Gauss Vortex Beam

R. Murugesan¹, N. Pasupathi², M. Udhayakumar³, M. Lavanya⁴, R. C. Saraswathi⁵,
K. B. Rajesh^{6*}

^{1,2}Department of Physics, Erode Arts College, Erode, TN, India.

^{*3,6}Department of Physics, Chikkanna Government Arts College, Tirupur, TN, India.

⁴Department of Physics, PSGR Krishnammal College for Women, Coimbatore, TN, India.

⁵Department of Physics, Government Arts College, Dharmapuri, TN, India.

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Abstract

Effect of astigmatism on the tight focusing properties of azimuthally polarized Lorentz–Gauss vortex beam is investigated numerically by the vector diffraction theory. Thus for non-vortex Lorentz beam the presence of astigmatism largely deform the focal structure and shifted the maximum intensity axially. However for the vortex Lorentz beam axially shifting with slightly deformation is observed. The author expect such a study is important in practical applications such as optical tweezers, laser printing and material processing.

Keywords: Azimuthally polarized; Lorentz–Gauss vortex beam; Vector diffraction theory.

1. INTRODUCTION

A Lorentz beam array is a good model to study a coherent diode laser array, and a detailed research of the propagation properties of a Lorentz beam array was presented by Zhou *et al.* 2010. They deduced the closed-form intensity distribution in the spatial frequency domain and investigated the effect of phase errors on the far-field intensity pattern. Recently, the Lorentz beams were also extended to the non-paraxial regime (Yu *et al.* 2010), in which propagation properties are illustrated and compared with numerical examples (Li *et al.* 2011). Recently, the Lorentz–Gauss beam has been introduced as a new kind of realizable beam (Roichman, *et al.* 2006). The Lorentz beam can be regarded as a special case of Lorentz-Gauss beams. With the spatial extension being the same, the angular spreading of a Lorentz-Gaussian distribution is higher than that of a Gaussian description (Unno *et al.* 2005). The presence of aberrations in the focusing system modifies the structure of the beam in the focal region, and this may cause serious problems in many applications (Visser *et al.* 1993; Liu *et al.* 2005). Structural modification in the focused structure of the doughnut beam due to aberration has also been

briefly mentioned by Willig *et al.* in the context of STED microscopy. The effect of primary aberrations on the focal structure of the beam has been investigated for a vortex-free beam (Visser *et al.* 1991; Kant 1993). Evaluated numerically the intensity distributions and encircled energy of focused singular beams at the focal plane in the presence of optical aberrations, such as the spherical aberration, defocusing, astigmatism and coma. More recently, the phase singularities of high numerical aperture dark-hollow Gaussian beams in the focal region were dealt with in Ref. (Biss *et al.* 2004), the tight focusing of an Lorentz-Gauss beam, we have investigated the effect of primary astigmatism on the focused structure of azimuthally polarized Lorentz Gauss vortex beam investigated by vector diffraction theory.

2. THEORY

Azimuthally polarized Lorentz-Gaussian beam focused through a high NA lens system focused. The analysis was performed on the basis of Richards and Wolf's vectorial diffraction method (Richards *et al.* 1959) widely used for high-NA lens system at arbitrary incident polarization. In the case

of the incident polarization, adopting the cylindrical coordinates r, z, ϕ and the notations (Youngworth *et al.* 2004), the electric field $E(r, z, \phi)$ in the vicinity of the focal region can be written as

$$E(r, \phi, z) = \begin{bmatrix} E_r \\ E_\phi \\ E_z \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{iA}{\pi} \int_0^\alpha \cos^2(\theta) \sin(\theta) A(\theta) \cos(\phi - \varphi) \exp(in\varphi) \exp[ik(z \cos \theta + r \sin \theta \cos(\phi - \varphi))] d\theta d\varphi \\ 0 \end{bmatrix} \rightarrow (1)$$

$n=0,1.$

Where $\alpha = \arcsin(NA)$, NA is the numerical aperture and n is the index of refraction between the lens and the sample. $A(\theta)$ Describes the Lorentz-Gaussian beam, this function is given by (Xiumin Gao *et al.* 2013).

$$A(\theta, \varphi) = \frac{C}{\omega_x \omega_y} \cdot \frac{1}{1 + \cos^2 \varphi \left(\sin^2 \theta / NA^2 \omega_x^2 \right)} \times \frac{1}{1 + \sin^2 \varphi \left(\sin^2 \theta / NA^2 \omega_y^2 \right)} \rightarrow (2)$$

Where $w_x = \omega_x/r_p$ is called relative beam waist in x coordinate direction. r_p is the outer radius of optical aperture in focusing system, f is focal length of the focusing system. NA is the numerical aperture of the focusing system

A_{st} denotes the wave front aberration function in the beam which can be expressed as (Kant, 1991;1995)

$$A_{Ast}(\theta) = \exp \left[I \cdot k \cdot A_{Ast} \left(\frac{\sin(\theta)}{\sin(\alpha)} \right)^2 \cos^2 \phi \right] \rightarrow (3)$$

Where the astigmatism coefficient A_{Ast} is in units of the wave length of the beam

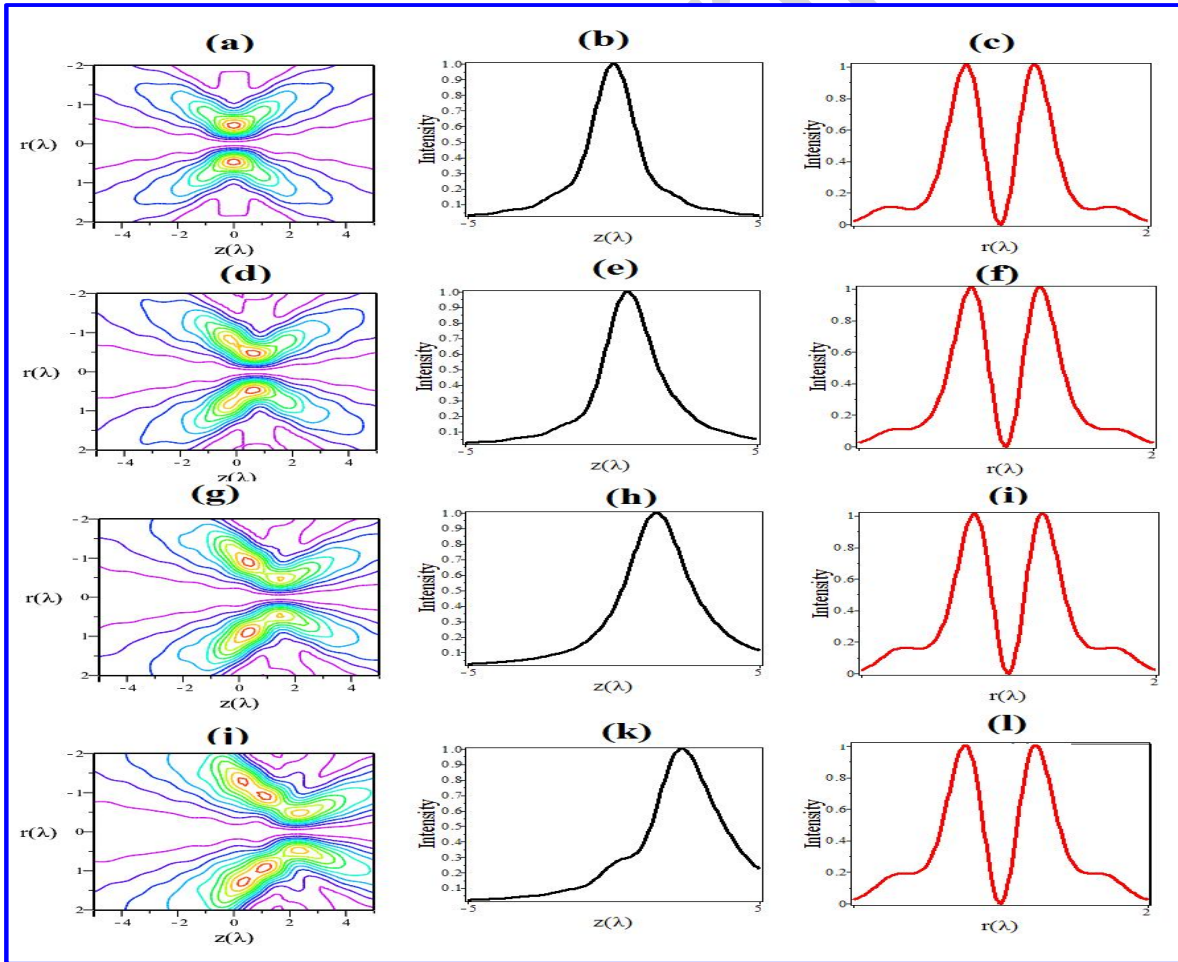


Fig. 1(a,b,c): Azimuthally polarized Lorentz-Gaussian beam, (d-l) azimuthally polarized Lorentz-Gaussian beam effect of astigmatism A_{st} for $NA = 0.95$, $w_x = 0.3$, $w_y = 0.3$, and $n = 0$. Fig.1(a,d,g,j) corresponding to $A_{st}=0\lambda, 0.5\lambda, 1\lambda$ and 1.5λ respectively. Fig.1(b,e,h,k) are corresponding intensity calculated in the radial axis. Fig.(c,f,i,l) are corresponding axial intensity distribution.

3. RESULT & DISCUSSION

We perform the integration of Eq. (1) numerically using parameters $\lambda = 1$, and $NA = 0.95$. Here, for simplicity, we assume that the refractive index $n = 1$. For all calculation in the length unit is normalized to λ and the energy density is normalized to unity. It is observed from the Fig.1 the increasing the astigmatism coefficient result in axial shifting of the focal hole, when $A_{st} = 0.5\lambda$. It is also noted that further increasing the astigmatism coefficient to 1.0λ further increasing shifting and deformation by off axial elongation in the tail part is observed. When $A_{st} = 1.5\lambda$ further shifting and elongation of tail part is observed.

Fig.2 shows the same as Fig.1 but for $w_x = 0.3$ and $w_y = 1.2$. It is noted that both axial shifting off axial elongation of tail part of is observed for the increasing the astigmatism coefficient however it is observed that deformation is much in this case when compared to the previous case.

Fig.3 shows the same as Fig.1 but for $w_x = 0.3$ and $w_y = 0.3, n = 1$. It is noted that increasing the astigmatism coefficient $A_{st} = 0.5\lambda$, slightly shifted to the focal spot in the axial direction. further increasing the astigmatism coefficient to 1.0λ results in radial elongation and axial shifting. The position of maximum intensity is observed at $1\lambda, 2\lambda$ and 2.4λ corresponding $A_{st} = 0.5\lambda, 1.0\lambda$ and 1.5λ .

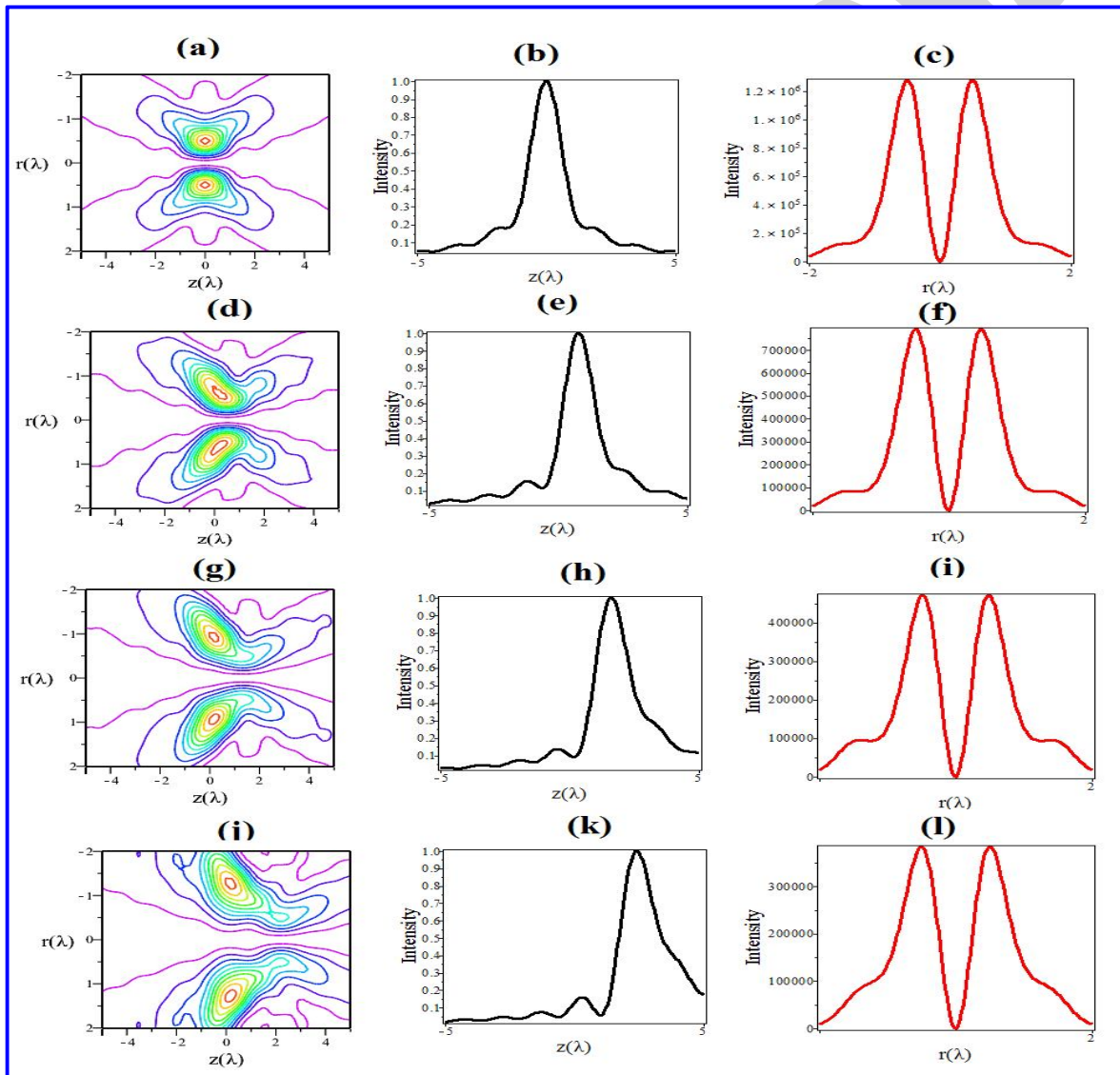


Fig. 2 (a,b,c): Radially polarized Lorentz–Gaussian beam, (d-l) azimuthally polarized Lorentz–Gaussian beam effect of astigmatism A_{st} for $NA = 0.95$, $w_x = 0.3$, $w_y = 1.2$, and $n = 0$. (a,d,g,j) corresponding to $A_{st} = 0\lambda, 0.5\lambda, 1\lambda$ and 1.5λ respectively. Fig.1(b,e,h,k) are corresponding intensity calculated in the radial axis. (e,f,i,l) are corresponding axial intensity distribution.

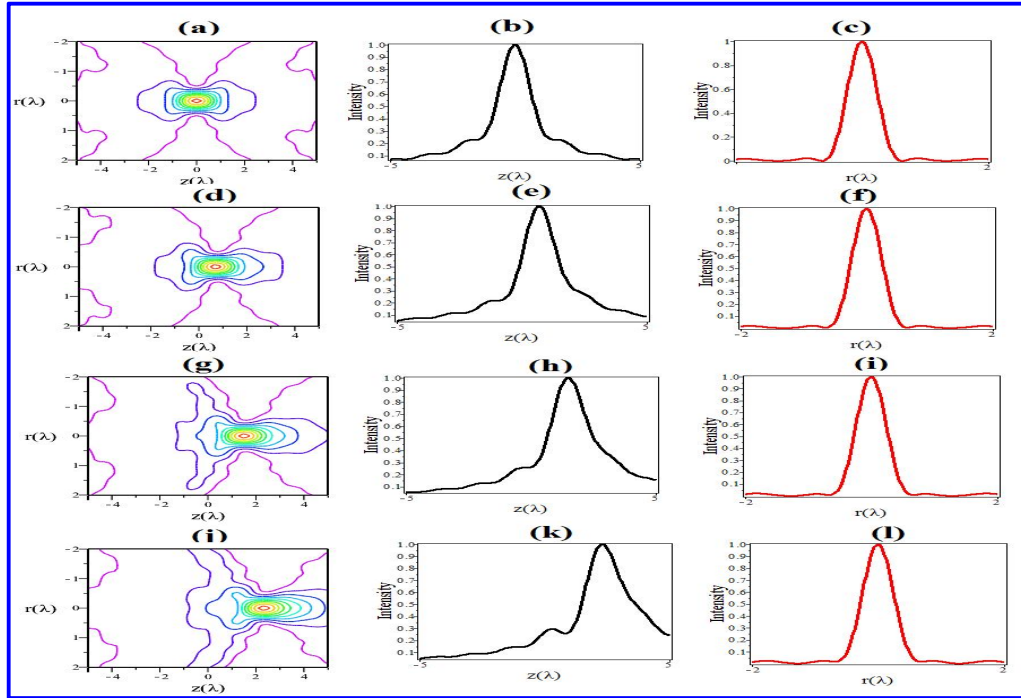


Fig. 3(a,b,c): Aazimuthally polarized Lorentz-Gaussian beam, (d-l) azimuthally polarized Lorentz-Gaussian beam effect of astigmatism A_{st} for $NA = 0.95$, $w_x = 0.3$, $w_y = 0.3$, and $n = 1$. Fig.(a,d,g,j) corresponding to $A_{st}=0\lambda, 0.5\lambda, 1\lambda$ and 1.5λ respectively.(b,e,h,k) are corresponding intensity calculated in the radial axis. (e,f,i,l) are corresponding axial intensity distribution.

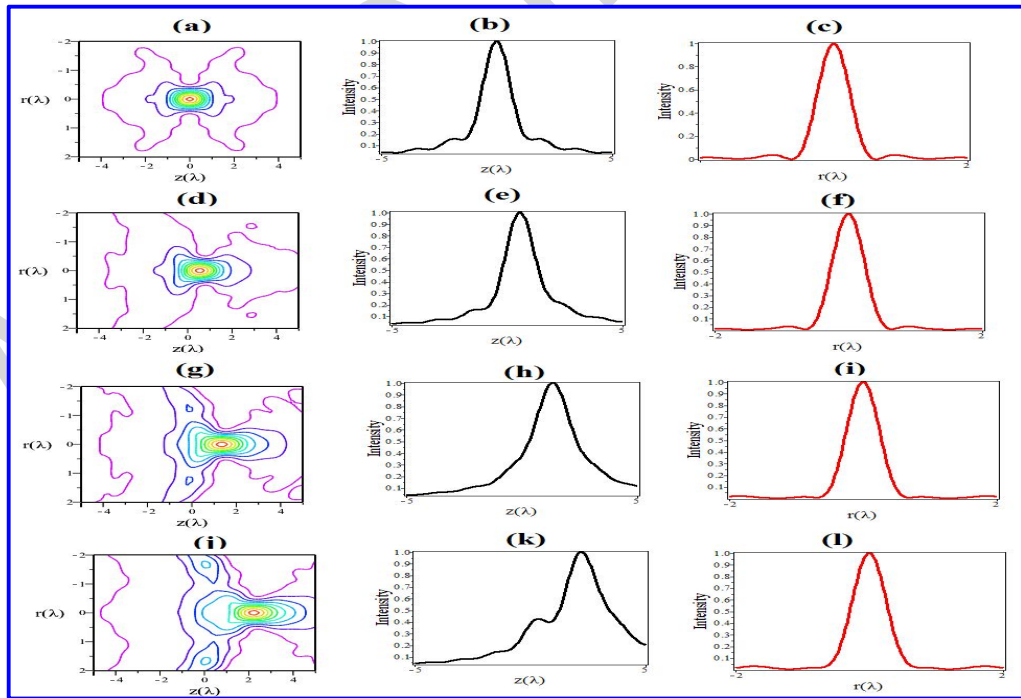


Fig. 4 (a,b,c): Azimuthally polarized Lorentz-Gaussian beam, (d-l) azimuthally polarized Lorentz-Gaussian beam effect of astigmatism A_{st} for $NA = 0.95$, $w_x = 0.3$, $w_y = 1.2$, and $n = 1$. Fig.(a,d,g,j) corresponding to $A_{st}=0\lambda, 0.5\lambda, 1\lambda$ and 1.5λ respectively.(b,e,h,k) are corresponding intensity calculated in the radial axis. (e,f,i,l) are corresponding axial intensity distribution.

Fig.4 shows the same as Fig.1 but for $n=1$. As if in the previous case position shift and axial elongation of the tail part is observed for the increasing astigmatism coefficient. Thus for non-vortex Lorentz beam the presence of astigmatism largely deform the focal structure and shifted the maximum intensity axially. However for the vortex Lorentz beam axially shifting with slightly deformation is observed.

4. CONCLUSION

Tight focusing properties of azimuthally polarized Lorentz–Gauss vortex beam with effect of astigmatism are investigated numerically by the vector diffraction theory. Thus for non-vortex Lorentz beam the presence of astigmatism largely deform the focal structure and shifted the maximum intensity axially. However for the vortex Lorentz beam axially shifting with slightly deformation is observed. Hence The author expect such a study is important in practical applications such as optical tweezers, laser printing and material processing

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